ANALYSIS OF THE MOTION OF SUSPENDED PARTICLES IN A ROTOR SEPARATOR

V. A. Markov and F. V. Prudnikov

UDC 60.074.1

We discuss a method for the calculation of the efficiency of a rotor-ring separator which includes a rotor made up of a set of plane rings and blades to form channels. The calculation reduces to an analysis of the motion engendered by the primary forces of the suspended particles in the channel between the blades and is based on the solution of differential equations by a numerical method. The research results have been generalized in the form of a relationship linking dimensionless complexes, this relationship suitable for engineering calculations.

Development of methods to separate vapor-gas mixtures from solid particles or from an atomized liquid, as well as the development of high-efficiency devices to achieve this separation, in addition to the design of contemporary technology and the modernization of existing equipment, is an urgent problem that is related both to questions of ecology and the efficient operation of heat and mass exchange equipment.

Choosing the method of separating suspended particles out of the gas phase is governed to a considerable extent by the unique features of the process and its regime parameters. Thus, for example, in cleansing industrial gases we encounter problems that relate to the fact that the volume of the gas is large, while the gas flow, as a rule, exhibits limited thrust. The separation equipment intended for these goals must therefore be both highly efficient and exhibit low hydraulic resistance. As is demonstrated by an analysis of the operation of various separators, the most promising are the separation units of the rotor type, whose primary advantage lies in their high productivity and in their compact design, since they can carry out simultaneously the functions of a separator and a ventilator. In terms of efficiency of operation, among separation units of this type the rotor devices with counterflow are preferable, since the gas being cleansed in these moves within the rotor from the periphery toward the center [1-3]. They consist of series connected separation and ventilation stages, rotating about a common shaft. The separation stage may be fashioned in the form of a rotor carrying a set of conical or plane rings or it may appear in the form of a rotating cylinder with channels to permit the passage of the gas flow.

Figure 1 shows a schematic diagram of a rotor-ring centrifugal separator which includes a rotor with a set of plane rings, with channels forming blades formed between these rings [4, 5]. The gas suspension enters the rotating channels and moves through them from the periphery of the rotor toward the center. The solid particles (or liquid droplets), on coming into contact with the blades, settle out on the blades and under the action of centrifugal forces are ejected out of the rotor against the wall forming the frame of the apparatus. Consequently, calculation of the efficiency with which suspended particles are separated out of the gas flow can be reduced to determination either of the geometric parameters of the rotor (the length of the blades, their number, etc.) for a given particle size and the number of rotor revolutions, or, conversely, on the basis of known geometric parameters the calculation is reduced to the determination of the required velocity of rotor rotation to accomplish the separation of particles of a specific dimension.

With a known blade length the number of rotor revolutions (or the number of blades) required for the particles, on passage through the channel, to separate out, i.e., to come into contact with the blade, can be calculated approximately by proceeding from the following assumptions. Let the velocity of the particle be equal to that of the gas flow and let it be directed along the blade. Then the continuity equation for the steady flow at a constant channel width is written in the form

$$rdr = w_0 R_o dt. \tag{1}$$

Having integrated (1) in limits from R_o to R_i and integrating time from 0 to t, we will determine the time t within which the particle covers the distance l, equal to $R_o - R_i$, i.e., the length of the channel (see Fig. 2):

$$t = \frac{(R_o - R_i)\,(R_o + R_i)}{2w_0 R_o} = \frac{l\,(1 + R_i/R_o)}{2w_0}. \tag{2}$$

Kirov Belorussian Technological Institute, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 60, No. 3, pp. 449-453, March, 1991. Original article submitted March 23, 1990.

 0022-0841/91/6003-0356$12.50

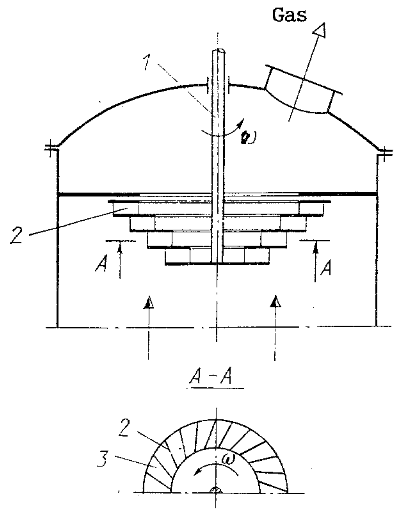

Fig. 1. Rotor-ring centrifugal separator: 1) rotor shaft; 2) blades; 3) channels.

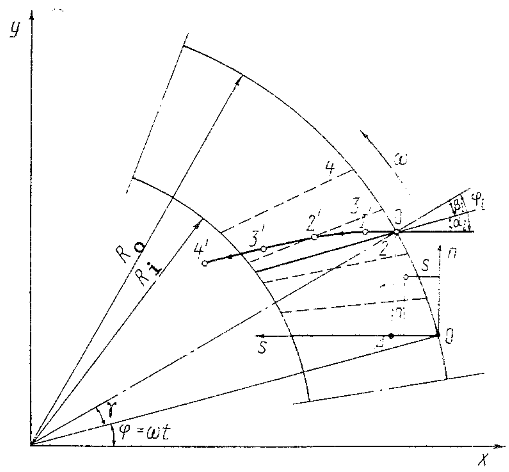

Fig. 2. Diagram for the calculation of particle motion trajectories.

Over this period of time the blade, as the rotor turns at an angular velocity ω, must shift thrugh an angle γ = 360°/m (m represents the number of blades) and cover the distance OO′, equal to OO′ = $2R_o \sin(\gamma/2)$. The time within which it accomplishes this displacement to position O′ is equal to

$$t = \frac{60 \sin(\gamma/2)}{\pi n_r}, \tag{3}$$

where n_r is the number of rotor revolutions, $\min^{-1}$.

When we equate (2) and (3), we obtain

$$n_r = \frac{38{,}2 w_0 \sin(\gamma/2)}{l(1 + R_i/R_o)}. \tag{4}$$

Thus, at values of n_r greater than those calculated in accordance with (4), the particles will settle out on the blades.

More precise relationships can be derived through an analysis of particle motion under the action of various forces. Let us examine the case in which the rotor axis is positioned vertically. A wheel with m blades, positioned at an angle β relative to the radius, rotates at a constant angular velocity ω. The particles are spherical in shape, and the velocity of the gas flow is directed along the blades. In order to solve the formulated problem, we will use the Lagrange equations [6]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = \Sigma Q. \tag{5}$$

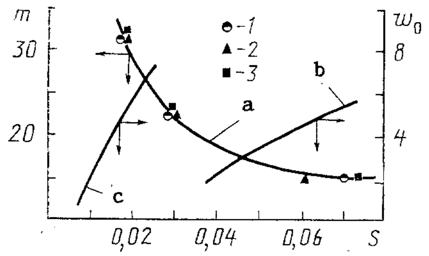

Fig. 1

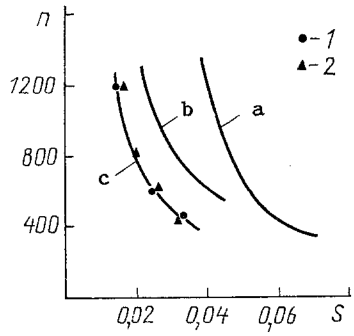

Fig. 2

Fig. 3. Influence exerted by the number m of blades and the gas velocity w_0 (m/sec) on the mean free path S (m) of the particles in the channel: a) n_r = 840 min^{-1}, $w_0 = v_s$ 5.0 m/sec, $v_n = 0$, $\beta = 0$, ρ_p = 1000 kg/m^3, 1) d = 0.5·10^{-3} m; 2) d = 0.5·10^{-4}; 3) d = 1.0·10^{-3}; b) d = 0.1·10^{-3} m, n_r = 840 min^{-1}, $w_0 = v_s$, v_n = 0, β = 0, m = 16, ρ_p = 1000 kg/m^3; c) d = 0.1·10^{-3} m, n_r = 840 min^{-1}, $w_0 = v_s$, v_n = 0, β = 0, m = 32, ρ_p = 1500 kg/m^3.

Fig. 4. Effect of the number of revolutions n_r (min^{-1}) on the mean free path S (m) of particles in the channel: a) d = 0.5·10^{-3} m, $w_0 = v_s$ = 3 m/sec, v_n = 0, β = 30°, m = 24, ρ_p = 1000 kg/m^3; b) d = 0.5·10^{-3} m, w_0 = v_s = 5 m/sec, v_n = 0, β = 0, m = 24, ρ_p = 1000 kg/m^3; c) $w_0 = v_s$ = 5 m/sec, β = 0, v_n = 0, m = 32; 1) d = 0.5·10^{-3} m, ρ_p = 800 kg/m^3; 2) d = 0.1·10^{-3} m, ρ_p = 1500 kg/m^3.

For the generalized coordinates q_i we will take the s and n axes. We will position the origin of the generalized coordinates at the outer radius of the wheel and we will direct these coordinates along the plate (the s axis) and perpendicular to it (the n axis). Equation (5) will then be written in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{n}}\right)-\frac{\partial T}{\partial n}=\Sigma Q_n,\quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{s}}\right)-\frac{\partial T}{\partial s}=\Sigma Q_s. \tag{6}$$

The origin of the nonmoving coordinates y and x will be perpendicular to the axis of rotation. The position of the particles at any instant of time t relative to the nonmoving coordinates can then be defined as

$$x = R_0 \cos(\omega t) - s\cos(\omega t-\beta) - n\sin(\omega t-\beta),$$
$$y = R_0 \sin(\omega t) - s\sin(\omega t-\beta) + n\cos(\omega t-\beta),$$

where $\omega t = \varphi$ and $\omega t - \beta = \alpha$ (see Fig. 2).

The kinetic energy T of the particle is equal to

$$T = \frac{m_p}{2}[\omega^2(R_0^2+s^2+n^2)+\dot{s}^2+\dot{n}^2+2R_0\omega\sin\beta(\dot{s}+\omega n)+$$
$$+2R_0\omega(\dot{n}-\omega s)\cos\beta+2\omega(\dot{s}n-s\dot{n})],$$

where m_p is the mass of the particle.

Having determined $\partial T/\partial n$, $\partial T/\partial s$, $\partial T/\partial \dot{n}$, $\partial T/\partial \dot{s}$, $d/dt(\partial T/\partial \dot{n})$ and $d/dt(\partial T/\partial \dot{s})$ and having substituted the derived values into (6), we obtain

$$\begin{aligned} m_p(\ddot{n}-2\dot{s}\omega-n\omega^2-R_0\omega^2\sin\beta)&=\Sigma Q_n,\\ m_p(\ddot{s}+\dot{n}\omega-s\omega^2+R_0\omega^2\cos\beta)&=\Sigma Q_s. \end{aligned} \tag{7}$$

The right-hand side of Eqs. (7) represents the sum of the projections of the generalized forces onto the n and s axes. In our case, these projections are of the force of hydrodynamic action, and they are, respectively, equal to:

$$P_n=\psi_n\frac{\pi d^2\rho_g v_n|\overline{v}|}{8},\quad P_s=\psi_s\frac{\pi d^2\rho_g[w(r)-v_s]|\overline{v}|}{8},$$

where $|\bar{v}|$ is the absolute value of the velocity, equal to $|\bar{v}| = \sqrt{v_n^2 + [w(r) - v_s]^2}$; ψ is the coefficient of resistance, defined from familiar expressions [7] as a function of the streamlining regime. It should be noted that the Archimedes force can be neglected, while the force of gravity, directed perpendicular to the n and s axes, for our case, is of no interest whatsoever, since the deflection of the particles in the vertical direction exerts no significant influence on separation.

After we have equated the derived values for the right-hand and left-hand sides of Eqs. (6) and having separated for mass, we finally obtain

$$\ddot{n} = 2\dot{s}\omega + (n + R_0 \sin\beta)\,\omega^2 - \psi_n k \dot{n} |\bar{v}|,$$
$$\ddot{s} = -2\dot{n}\omega + (s - R_0\cos\beta)\,\omega^2 + \psi_s k\,[w(r) - \dot{s}]|\bar{v}|, \tag{8}$$

where $k = 3\rho_g/(4\rho_p d)$.

We will determine the initial conditions by analyzing the operation of the separator as a function of the influencing factors. It is obvious that the particles at the inlet to the channels of the wheel can move in a variety of directions relative to the blade. If we bear in mind the small particles whose velocity is nearly that of the gas flow, then their direction may be assumed to coincide with that of the gas flow. The large particles "lag" behind the gas velocity, i.e., they have a lower velocity. Consequently, the smaller the particle and the higher its velocity, the greater the "mean-free path" of the particle to contact with the blade. Moreover, the location (coordinate) of the particle inlet also affects this distance, and the greater the distance (the greater the coordinate n) of the particles from the "incident" wheel blade, the greater the distance that it covers without coming into contact with the blade. Thus, from the standpoint of separation the poorest initial conditions will be found at particle velocities equal to the gas velocity ($v_s = w_0$), an initial velocity $v_n \neq 0$, and initial values for s_0 and n_0 equal to

$$s_0 = 2R_0 \sin(\gamma/2) \sin\left(\frac{\gamma}{2} - \beta\right), \quad n_0 = R_0 \sin\gamma \text{ when } \beta \geqslant 0$$

and

$$s_0 = -2R_0 \sin(\gamma/2) \sin(\beta - \gamma/2),$$
$$n_0 = 2R_0 \sin(\gamma/2) \cos(\beta - \gamma/2) \text{ when } \beta < 0.$$

System of equations (8) was calculated by the Runge–Kutta–Fel'berg numerical method for a variety of regime and structural wheel parameters. The gas velocity w_0 in this case varied within limits of 1-10 m/sec, the particle diameters d varied within limits of $5 \cdot 10^{-5}$-$1 \cdot 10^{-3}$ m, the v_s/w_0 and $|v_n|/w_0$ ratios varied within limits of 0-1 and the particle density ρ_p fell within limits of 800-1500 kg/m^3.

The numerals 1-4 in Fig. 2 show the position of the blades, while the numerals 1'-4' identify the particles after identical intervals of time. It follows from Figs. 3 and 4, which show some of the computational results, that the number of rotor revolutions, as well as the number of blades, significantly affect the required blade length. At the same time, the particle diameters and the particle densities for the adopted initial conditions have little effect on the mean-free path. This latter circumstance is apparently explained by the fact that the particles fail "to accelerate" over a small segment as a consequence of the gas flow or to "lag" behind the flow as the velocity of the gas changes over the radius.

Results from calculations on the basis of (8) were approximated in the form of a relationship of dimensionless complexes convenient for engineering computations of the channel length S (m), as well as for known values of the latter, and such other parameters, such as, for example, m, ω, or β:

$$S = 22{,}4\,\mathrm{Re}^a \left(\frac{v_s}{\omega R_0 \cos\beta}\right)^b \left(\frac{|v_n|}{w_0}\right)^c m^f, \tag{9}$$

where $a = 0.044$, $b = 0.76$, $c = -0.1$, and $f = -1.82$.

The mean relative error in the computed values from (9) and (8) for this range of change in the parameters does not exceed 20%. The values calculated in accordance with (9) are somewhat larger than the quantities determined from the approximation function (4), and for the calculations we consequently should recommend expression (9).

The computational results were used in developing a design of a rotor separator for the dry cleansing of gases discharged from the nepheline–antipyrene operation at the Gomel' Chemical Plant.

NOTATION

d, particle diameter; m, number of blades; m_p, particle mass; n and s, generalized coordinate axes; $\dot{n}$ and $\dot{s}$, generalized velocities; n_r, number of rotor revolutions; w_0 and w(r), gas velocities at the inlet to the channel and the instantaneous velocity of the radius; v_n and v_s, projections of particle velocities onto the n and s axes, respectively; $|v_n|$, projection of particle velocity onto the n axis in terms of absolute magnitude; R_0 and R_i, outer and inner wheel radii; y and x, nonmoving coordinate axes; T, kinetic energy of the particle; Q_n and Q_s, projection of the generalized forces onto the n and s axes, respectively; ω, angular velocity; ψ, resistance factor; ρ_p and ρ_g, density of particle and gas, respectively; μ_g, gas viscosity; $Re = w_0 d\rho_g/\mu_g$, Reynolds number.

LITERATURE CITED

1. B. Brettschneider and J. Kurfuerst, *Protecting the Water Basin from Contamination. Technology and Control* [Russian translation], A. F. Tubolkhin (ed.), Leningrad (1989).
2. P. A. Kouzov, A. D. Mal'gin, and G. M. Skryabin, *Removal of Dust from Gases and Air in the Chemical Industry* [in Russian], Leningrad (1982).
3. N. N. Zhenishek, *Rotational Dust Removal* [in Russian], Moscow (1958).
4. "Centrifugal separator," USSR Inventor's Certificate, No. 521,911: MKI^2 V 01 D 45/12.
5. "Centrifugal foam extinguisher," USSR Inventor's Certificate, No. 578,330: MKI^2 S 12 V 1/18.
6. S. M. Targ, *A Brief Course of Theoretical Mechanics* [in Russian], Moscow (1967).
7. A. G. Kasatkin, *Fundamental Processes and the Apparatus of Chemical Technology* [in Russian], Moscow (1971).

USING THE REVERSABILITY OF THE PELTIER EFFECT TO REDUCE THE HEAT-SCATTERING SURFACES OF THERMAL COOLING BATTERIES

E. K. Iordanishvili

UDC 537.324:536.24

We propose and analyze a method for significant reduction in the area of heat scattering surfaces in thermal cooling batteries, where the latter are used to cool small-scale objects.

In view of the increasing scale on which thermoelectric cooling batteries are used presently to cool small-scale objects in radio- and microelectronics, considerable importance has been ascribed to the dissipation of heat from the hot junctions of thermal batteries. In engineering practice, with this purpose in mind, use is often made of radioelectronic equipment (REE) design elements, and here, in a number of cases, owing to the small dimensions of the apparatus itself, such surfaces turn out to be overheated, and this in turn leads to an elevation in the temperature of the object to be cooled, i.e., to a reduction in the efficiency of the cooling process.

In the present study we propose and analyze a method of activating the heat-scattering process by altering (elevating) the temperature of an artificially generated heat-scattering surface of smaller area, while retaining a lower temperature in the object being cooled. The basic diagram of the proposed method is illustrated in Fig. 1. In Fig. 1a we see illustrated the situation which actually prevails in the cooling of REE elements. The REE element is cooled by means of a single-cascade thermal battery 2, removing heat to the bounded upper surface of the device (a radiator), and here, owing to the inadequate size of the heat-scattering surface 1 and 3, a rather substantial drop in temperature is produced between the radiator and the ambient medium, which in the final analysis reduces the capabilities of the REE element. Within the scope of the proposed method, this same surface (Fig. 1b) is connected to the second thermal cooling battery 4 through its cold junction. Its hot junction in this case is oriented into open space and fitted with radiator 5 to dissipate heat at a higher temperature potential. If it is possible to prove that the area of radiator 5 in such a scheme is significantly smaller than heat-scattering surfaces 1 and 3, the problem may be regarded as having been solved. Particular attention should be devoted here to the fact that the smaller area 5 replaces the larger area 3, thus achieving the same (and possibly a lower) level of cooling for the REE object.

Kalinin Polytechnical Institute, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 60, No. 3, pp. 453-458, March, 1991. Original article submitted December 28, 1989.

0022-0841/91/6003-0360$12.50